Electromagnetic energy within magnetic spheres

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Consider that an incident plane wave is scattered by a homogeneous and isotropic magnetic sphere of finite radius. We determine, by means of the rigorous Mie theory, an exact expression for the time-averaged electromagnetic energy within this particle. For magnetic scatterers, we find that the value of the average internal energy in the resonance picks is much larger than the one associated with a scatterer with the same nonmagnetic medium properties. This result is valid even, and specially, for low size parameter values. Expressions for the contributions of the radial and angular field components to the internal energy are determined. For the analytical study of the weak absorption regime, we derive an exact expression for the absorption cross section in terms of the magnetic Mie internal coefficients. We stress that although the electromagnetic scattering by particles is a well-documented topic, almost no attention has been devoted to magnetic scatterers. Our aim is to provide some new analytical results, which can be used for magnetic particles, and emphasize the unusual properties of the magnetic scatters, which could be important in some applications.

I. INTRODUCTION

The research in magneto-optics, both theoretical and experimental, has been mainly devoted to the study of magnetic properties of thin films. Magneto-optical effects are characterized by the change in the state of light polarization in the presence of magnetic materials, both in transmission (Faraday effect) and reflection (Kerr effect). Brillouin light scattering technique allows the investigation of spin waves in magnetic films and layered structures through the light scattering by magnons. Here, we are concerned with another feature in the magneto-optics research: the electromagnetic (EM) scattering by magnetic particles [1–5]. Although the EM scattering by particles is a well-documented topic [6–10], little attention has been given to the case of EM scattering by magnetic particles. Recently, it has been a growing interest on photonic band gap materials (PBGs) made of ferromagnetic materials, like soft ferrites, at microwave or radio frequencies [11–14]. Other important applications involving magnetic materials, such as microwave filters, metamaterials, high density magnetic recording media, have been reported [15, 16]. here, the approach we follow is the classical one for single Mie scattering [6, 7, 9, 10], in which no applied external field is considered.

The EM radiation scattering by magnetic spheres is described on the basis of the Mie theory, in which an incident plane wave, with wavenumber k, is scattered by a homogeneous sphere of radius a. We assume that both the scatterer and the medium are nonmagneto-optical active and that the incident radiation is a vetorial wave. In general, the bulk of analysis takes place in far-field approximation, ignoring the evanescence and the internal fields in the scattering center [6, 7, 9]. The interest, therefore, lies in the behavior of the scattered fields and all the quantities of interest to describe EM scattering by spherical particles, such as cross sections and the anisotropy factor $\langle \cos\theta \rangle$ (i.e., the mean value of the cosine of the scattering angle θ), can be expressed in terms of the Mie coefficients a_n and b_n [9]. For magnetic scatterers, in particular, a_n and b_n have been obtained by Kerker et al. [1]. Nevertheless, here, as in the original work of Bott and Zdunkowski [17] for nonmagnetic spheres, the internal fields in Mie single scattering gain special attention and some related quantities are studied.

Bott and Zdunkowski [17] present the exact and approximate analytical expressions for the time-averaged EM energy within a dielectric sphere. The calculations have been anchored on the rigorous Mie theory, and the expressions have been derived, as usual, with the assumption of equality between the magnetic permeability tensors of medium and particle. This configuration is denominated *nonmagnetic scattering* [1]. It is pointed out in [17] that those calculations are of importance for the study of photochemical reactions within atmospheric water spheres.

The aim of this paper is to provide a detailed description of the time-averaged EM energy within magnetic

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particles (assumed to be spherical), emphasizing their unusual properties, which in turn could be explored in as microwave filters and PBGs [15] or in the search of photon localization in the multiple scattering regime [4, 5]. In Sec. II is presented a brief resume about the construction of the exact solution in the single magnetic Mie scattering and its principal analytical results. Both the EM internal fields and the magnetic Mie coefficients are presented. The determination of the exact expression for the time-averaged EM energy within a magnetic scatterer is shown in Sec. III. The problem symmetry allows us to express separately the contribution of the radial and angular components to the average internal energy. A new expression for the absorption cross section in the magnetic case is determined. To validate our expressions, for instance, we determine the same particular relations studied in [17]. Special attention is paid to our approach concerning the differences to [17]. Finally, we present some numerical results in Sec. IV. We compare the magnetic and nonmagnetic scattering. The basic relations involving the Bessel and associated Legendre functions are presented in the Appendix A. Those expressions are important to the calculation of the quantities related to the time-averaged internal energy. In the Appendix B, some classical limiting cases are considered and we give a set of approximated magnetic Mie coefficients.

II. ANALYTICAL CALCULATION OF SCATTERING QUANTITIES

To deal with EM wave scattering by a single particle embedded in a medium, one must assume some special features for the medium and the incident wave. Among these assumptions, the particle is considered isolated in an infinite medium, which allows one to ignore the effect of multiple scattering [6, 7]. Both particle and medium are considered linear, homogeneous, and isotropic, having inductive capacities (ϵ_1, μ_1) and (ϵ, μ), respectively. Thereby, once we assume the media are nonmagneto-optic active, those tensors, respective to magnetic permeability (μ) and electric permittivity (ϵ), can be expressed by a scalar quantity times an unitary tensor. In particular, it is assumed that there are absorptive components within the scatterer, so the quantities ϵ_1 and μ_1 are complex.

The incident radiation is considered plane, monochromatic, and polarized complex EM wave, which is expressed as

$$\mathbf{E}_i(\mathbf{r}, t) = \mathbf{E}_0 \exp\left[\imath\left(\mathbf{k}\cdot\mathbf{r} - \omega t\right)\right], \qquad (1)$$

with wave amplitude $\mathbf{E}_0 = E_0\mathbf{e}_x$, wave vector $\mathbf{k} = k\mathbf{e}_z$ and angular frequency $\omega = 2\pi/\lambda$, where λ is the wavelength. Due to the spherical symmetry of the scattering center, there is no loss of generality taking the electric field polarized on the x axis direction. Also, the linearity of the macroscopic Maxwell equations and Fourier theory allow one to generalize this monochromatic case to a polychromatic one [6].

The incident, scattered and internal vector waves have the same angular frequency ω, once we are not accounting for possible energy variations in the interaction with the scatterer. Thus, quantum fluctuations such as in Raman scattering is neglected, and a classical description is adopted [6, 7].

In the rigorous Mie theory it is quite common to assume the equality between the magnetic permeability tensors of the particle and medium. This consideration ignores the most general case in which these complex tensors are different. The absolute value of the magnetic permeability μ_1 can assume values much larger than μ, as in the case of soft ferromagnetic particles in the microwave range, for instance [2, 11]. In this present work, the Mie coefficients are recalculated in this general case, referred to as *magnetic scattering* [1, 3–5], and the associated Mie coefficients of the internal fields, which have not been studied so far, are presented. The expressions here obtained are valid for a wide class of soft ferrites with magnetic loss. The assumption of the isotropic magnetic permeability (and electric permittivity) allows one to solve the scattering problem in a simple way. However, to lower the magnetic loss of these magnetic materials, it is usually to consider them in presence of an applied external magnetic field [11–13, 15, 16]. In this situation, the relative magnetic permeability is anisotropic and its tensor elements are depends on this externally applied field.

From the Maxwell theory, we have that a time-harmonic EM field $(\mathbf{E}, \mathbf{H})$ in an homogeneous, isotropic and linear medium must satisfy the vectorial Helmholtz equation $[\nabla^2 + k^2]\mathbf{E} = \mathbf{0}$, $[\nabla^2 + k^2]\mathbf{H} = \mathbf{0}$, where $k^2 = -\epsilon\mu\partial_t^2 = \omega^2\epsilon\mu$, and be divergence-free null: $\nabla\cdot\mathbf{E} = \nabla\cdot\mathbf{H} = 0$. The quantity $|\mathbf{k}| = k$ is the wavenumber and it is related to the travelling wave. In addition, $\mathbf{E}$ and $\mathbf{H}$ are not independent: $\nabla\times\mathbf{E} = \imath\omega\mu\mathbf{H}$, $\nabla\times\mathbf{H} = -\imath\omega\epsilon\mathbf{E}$. To simplify the resolution of these equations, we build solutions which are dependent of a scalar function ψ, called a generating function for the vector harmonics [6, 7]. In this particular case, once the symmetry of the problem is spherical, the solutions of the equations above are the spherical vector harmonics, expressed by $\mathbf{M} = \nabla\times(\mathbf{r}\psi)$ and $\mathbf{N} = \nabla\times\mathbf{M}/k$. The imposition of the vector harmonics are solutions of the Maxwell equations implies that $[\nabla^2 + k^2]\psi = 0$. Thus, the problem of the scattered waves by a spherical particle resumes to solve this scalar Helmholtz equation in spherical coordinates.

Another way to tackle this problem without considering the vector harmonics employs the Hertz potential [9, 18]. However, we prefer to adopt the same framework of Bohren and Huffman [6], in which the plane waves are directly expanded in terms of spherical vector harmonics. The solution of the scalar Helmholtz equation is $\psi_{nm}(kr, \cos\theta, \phi) = z_n(kr)P_n^m(\cos\theta)\exp(\imath m\phi)$, where $z_n(kr)$ is a generic Bessel spherical function and $P_n^m(\cos\theta)$ are associated Legendre functions, n natural

and m integer. By means of ψ_{nm}, we can readily derive the spherical vector harmonics above defined. From the expansion of the incident fields in terms of $\mathbf{M}_{nm}$ and $\mathbf{N}_{nm}$, we find that only $m=1$ contributes to this new representation due to the spherical symmetry of the scatterer [6, 7]. Using the boundary conditions of this problem, we can express the internal and the scattered fields in terms of spherical vector harmonics. In special, the coefficients of these expansions are referred to as the *Mie coefficients*. They provide the information about the interaction between the incident wave and the spherical particle. Explicitly, the boundary condition ($r=a$) is expressed by $(\mathbf{E}_i+\mathbf{E}_s-\mathbf{E}_1)\times\mathbf{e}_r = (\mathbf{H}_i+\mathbf{H}_s-\mathbf{H}_1)\times\mathbf{e}_r = \mathbf{0}$, where 1 is the index related to the particle (internal fields), and i and s refer to the incident and scattered fields, respectively; $\mathbf{e}_r$ is the radial unity vector in the polar spherical coordinate system.

A. Internal fields

Assuming the incident EM wave is polarized in the $\mathbf{e}_x$ direction, and the scattering center is placed at the origin of the coordinate system, we obtain an expression for the expansion of this field in terms of spherical harmonics. Imposing on the boundary between the sphere and the surrounding medium the continuity of the EM fields $-$ in fact, their (electrical and magnetic) tangential components $-$, expressions are determined for the internal and scattered fields [6, 7, 9].

Using the same notation of Bohren and Huffman [6], we can give the components of the electric and magnetic vectors, $\mathbf{E}_1$ and $\mathbf{H}_1$, respectively, of the interior field in a spherical coordinate system (r,θ,ϕ) by

$$
\begin{aligned}
E_{1r} &= \frac{-\imath\cos\phi\sin\theta}{\rho_1^2}\sum_{n=1}^{\infty}E_n d_n\psi_n(\rho_1)n(n+1)\pi_n \ , \\
E_{1\theta} &= \frac{\cos\phi}{\rho_1}\sum_{n=1}^{\infty}E_n\left[c_n\pi_n\psi_n(\rho_1)-\imath d_n\tau_n\psi_n'(\rho_1)\right] \ , \\
E_{1\phi} &= \frac{\sin\phi}{\rho_1}\sum_{n=1}^{\infty}E_n\left[\imath d_n\pi_n\psi_n'(\rho_1)-c_n\tau_n\psi_n(\rho_1)\right] \ ; \\
H_{1r} &= \frac{-\imath k_1}{\omega\mu_1}\frac{\sin\phi\sin\theta}{\rho_1^2}\sum_{n=1}^{\infty}E_n c_n\psi_n(\rho_1)n(n+1)\pi_n \ , \\
H_{1\theta} &= \frac{k_1}{\omega\mu_1}\frac{\sin\phi}{\rho_1}\sum_{n=1}^{\infty}E_n\left[d_n\pi_n\psi_n(\rho_1)-\imath c_n\tau_n\psi_n'(\rho_1)\right] \ , \\
H_{1\phi} &= \frac{k_1}{\omega\mu_1}\frac{\cos\phi}{\rho_1}\sum_{n=1}^{\infty}E_n\left[d_n\tau_n\psi_n(\rho_1)-\imath c_n\pi_n\psi_n'(\rho_1)\right] \ .
\end{aligned}
$$

with $\rho_1 = k_1 r$, $E_n = \imath^n E_0(2n+1)/[n(n+1)]$, π_n and τ_n are angular functions defined in the appendix A 2 and $\psi_n(\rho_1) = \rho_1 j_n(\rho_1)$ is a Ricatti-Bessel function. The functions c_n and d_n are the internal magnetic Mie coefficients, which are presented in the section below. We outline that, because of the notation (units in the SI and time factor) here adopted, the internal EM field $(\mathbf{E}_1,\mathbf{H}_1)$ is not the same presented in [17].

B. Magnetic Mie coefficients

If we do not assume the equality between μ and μ_1 on the problem boundary condition, we can determine the *magnetic Mie coefficients* for the scattering (a_n and b_n, obtained by Kerker at al.) and internal (c_n and d_n) fields [1–5]. Explicitly,

$$
\begin{aligned}
a_n &= \frac{\widetilde{m}\psi_n(mx)\psi_n'(x)-\psi_n(x)\psi_n'(mx)}{\widetilde{m}\psi_n(mx)\xi_n'(x)-\xi_n(x)\psi_n'(mx)} \ , &(2)\\
b_n &= \frac{\psi_n(mx)\psi_n'(x)-\widetilde{m}\psi_n(x)\psi_n'(mx)}{\psi_n(mx)\xi_n'(x)-\widetilde{m}\xi_n(x)\psi_n'(mx)} \ , &(3)\\
c_n &= \frac{m\imath}{\psi_n(mx)\xi_n'(x)-\widetilde{m}\xi_n(x)\psi_n'(mx)} \ , &(4)\\
d_n &= \frac{m\imath}{\widetilde{m}\psi_n(mx)\xi_n'(x)-\xi_n(x)\psi_n'(mx)} \ , &(5)
\end{aligned}
$$

with the assumption that the function domains are restricted in such a manner that the denominators do not vanish. The quantity $x = ka$ is the size parameter of the spherical particle, being a its radius and $k = |\mathbf{k}|$ the wavenumber of incident and scattered waves, and $\xi_n(x) = x[j_n(x)+\imath y_n(x)]$ is the Ricatti-Hankel function of first kind. In addition, $m = (\mu_1\epsilon_1/\mu\epsilon)^{1/2}$ is the relative refraction index and $\widetilde{m} = (\mu\epsilon_1/\mu_1\epsilon)^{1/2}$ is the relative impedance between the media. For $\mu=\mu_1$, then $\widetilde{m}=m$ and the usual expressions for the Mie coefficients (2-5) are recovered [6, 7, 9].

There are some notation differences between this work and the one presented by Bott and Zdunkowski [17]. Here, we use the same framework of Bohren and Huffman [6], which have treated the scattering problem of light by means of International System of Units (SI), and have adopted $\exp(-\imath\omega t)$ as time-harmonic dependency for the fields. Otherwise, [17] have used the same notation as van de Hulst [7], which has dealt with the scattering problem in the Gaussian System of Units, and has adopted $\exp(\imath\omega t)$ as time-harmonic dependency. These approach differences appear explicitly in the choice of the Hankel functions, which is strictly associated with the asymptotic limit for the scattered fields (the well-known far-field approximation), and consequently determines the dependencies of the Mie coefficients. Another difference between these representations is related to the signal of the imaginary part of the relative refraction index $m = m_r + \imath m_i$, which is positive in the framework we have chosen [6, 7].

III. TIME-AVERAGED ELECTROMAGNETIC ENERGY

For a linear, homogeneous, and isotropic medium, the classical theory of electromagnetism provides an expres-

sion for the time-averaged EM energy as an integral of the component intensities within the volume under analysis. In the case of a spherical particle with radius a and internal complex EM field $(\mathbf{E}_1, \mathbf{H}_1)$, we have the following relation [17, 19]:

$$\begin{aligned} W(a) &= \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \mathrm{d}(\cos\theta) \int_0^a \mathrm{d}r r^2 \\ &\times \mathrm{Re}\bigg[\frac{\epsilon_1}{4} \left(|E_{1r}|^2 + |E_{1\theta}|^2 + |E_{1\phi}|^2 \right) \\ &+ \frac{\mu_1}{4} \left(|H_{1r}|^2 + |H_{1\theta}|^2 + |H_{1\phi}|^2 \right) \bigg] . \end{aligned} \quad (6)$$

In Eq. (6), with respect to the field representations, we outline that the permutation between a definite integral and a sum of an infinite series is not trivial. In the following calculations, we are not concerned about showing explicitly each one of the simplifications; we just assume that the function series related to the field intensities converge uniformly in the domain $0 < r \le a$, $0 \le \theta \le \pi$, $0 < \phi \le 2\pi$. Obviously, this mathematical condition is in agreement that the energy within a finite sphere is also finite.

A. Electric and magnetic internal fields

Looking closely to the definition of Eq. (6), one can ask about the contribution to the total internal energy associated with electric and magnetic fields separately, or even about the contribution of their fields components in spherical coordinates (r, θ, ϕ) to this average energy. These questions have not addressed by Bott and Zdunkowski in their [17] paper.

1. Radial component

From Eq. (6), we obtain that the contribution of the radial component associated with the electric field to the internal energy is given by

$$\begin{aligned} W_{rE}(a) &= \frac{\mathrm{Re}(\epsilon_1)}{4} \int_{-1}^{1} \mathrm{d}(\cos\theta) \int_0^{2\pi} \mathrm{d}\phi \int_0^a \mathrm{d}r r^2 \big| E_{1r} \big|^2 \\ &= \frac{\pi}{4} \mathrm{Re}(\epsilon_1) \int_0^a \mathrm{d}r \sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} E_n E_{n'}^* j_n(\rho_1) j_{n'}^*(\rho_1) \frac{d_n d_{n'}^*}{|k_1|^2} \\ &\times n n'(n+1)(n'+1) \underbrace{\int_{-1}^{1} \mathrm{d}(\cos\theta) \sin^2\theta \pi_n \pi_{n'}}_{\text{Eq. (A5)}} \\ &= \frac{\pi}{2} |E_0|^2 \frac{\mathrm{Re}(\epsilon_1)}{|k_1|^2} \sum_{n=1}^{\infty} n(n+1)(2n+1) \\ &\times |d_n|^2 \int_0^a \mathrm{d}r |j_n(\rho_1)|^2 . \end{aligned} \quad (7)$$

Proceeding in the same way, one derives an analogous expression for the radial component related to the magnetic field:

$$\begin{aligned} W_{rH}(a) &= \frac{\pi}{2} |E_0|^2 \frac{\mathrm{Re}(\mu_1^{-1})}{\omega^2} \sum_{n=1}^{\infty} n(n+1) \\ &\times (2n+1) |c_n|^2 \int_0^a \mathrm{d}r |j_n(\rho_1)|^2 . \end{aligned} \quad (8)$$

An important point to be noted here is that the integral above cannot be simplified by means of Eq. (A1) and recurrence relations presented in Appendix A 1.

2. Angular components

Because of spherical symmetry of the system, it is not possible to write the contributions of the angular and azimuthal components to internal energy separately. If one tries to do that, the necessary relations to simply the *double sums*, as exemplified in Eq. (7), do not appear. Fortunately, if one considers both (θ, ϕ) contributions to internal energy, these relations are not lost. Thus, using the relations (A3) and (A4) from Appendix A 2 and the first term of Eq. (6), it follows that the time-averaged energy associated with angular components of the electric field is expressed by

$$\begin{aligned} [W_{\theta E} + W_{\phi E}](a) &= \frac{\mathrm{Re}(\epsilon_1)}{4} \int_{-1}^{1} \mathrm{d}(\cos\theta) \int_0^{2\pi} \mathrm{d}\phi \\ &\times \int_0^a \mathrm{d}r r^2 \left(|E_{1\theta}|^2 + |E_{1\phi}|^2 \right) \\ &= \frac{\pi}{2} |E_0|^2 \frac{\mathrm{Re}(\epsilon_1)}{|k_1|^2} \sum_{n=1}^{\infty} (2n+1) \\ &\times \int_0^a \mathrm{d}r \left(|c_n \psi_n(\rho_1)|^2 + |d_n \psi_n'(\rho_1)|^2 \right) . \end{aligned} \quad (9)$$

Similarly, for the magnetic field we obtain

$$\begin{aligned} [W_{\theta H} + W_{\phi H}](a) &= \frac{\pi}{2} |E_0|^2 \frac{\mathrm{Re}(\mu_1^{-1})}{\omega^2} \sum_{n=1}^{\infty} (2n+1) \\ &\times \int_0^a \mathrm{d}r \left(|d_n \psi_n(\rho_1)|^2 + |c_n \psi_n'(\rho_1)|^2 \right) . \end{aligned} \quad (10)$$

Here, the same problem of the expression $W_r(a)$ arises. Whereas the integral of Ricatti-Bessel function is only another way to write Eq. (A1), the second integral above cannot be simplified.

B. Time-averaged internal energy

From the expressions obtained in the previous section for each one of the internal field components, we can calculate the total time-averaged energy inside the sphere.

For the internal electric field, the expression is

$$\begin{aligned} W_E(a) &= W_{rE}(a) + [W_{\theta E} + W_{\phi E}](a) \\ &= \frac{3}{4} W_0 \mathrm{Re}(m\widetilde{m}) \sum_{n=1}^{\infty} \left\{ (2n+1)|c_n|^2 \mathcal{I}_n(y) \right. \\ &\quad \left. + |d_n|^2 \left[n\mathcal{I}_{n+1}(y) + (n+1)\mathcal{I}_{n-1}(y) \right] \right\} , \end{aligned} \tag{11}$$

where

$$\mathcal{I}_n(y) = \frac{1}{a^3} \int_0^a \mathrm{d}r r^2 \left| j_n(\rho_1) \right|^2 \tag{12}$$

is given by Eq. (A1) and W_0 denotes the time-averaged EM energy of a sphere with radius a having the same EM properties of the surrounding medium:

$$W_0 = \frac{2}{3} \pi a^3 |E_0|^2 \epsilon \ . \tag{13}$$

For sake of simplicity, the dependence of $\mathcal{I}_n$ with respect to $y^* = m^* ka$, like the case of function $W = W(a, y, y^*)$, is omitted.

In the same way, for the internal magnetic field, the average internal energy is given by

$$\begin{aligned} W_H(a) &= W_{rH}(a) + [W_{\theta H} + W_{\phi H}](a) \\ &= \frac{3}{4} W_0 \mathrm{Re}(m\widetilde{m}^*) \sum_{n=1}^{\infty} \left\{ (2n+1)|d_n|^2 \mathcal{I}_n(y) \right. \\ &\quad \left. + |c_n|^2 \left[n\mathcal{I}_{n+1}(y) + (n+1)\mathcal{I}_{n-1}(y) \right] \right\} . \end{aligned} \tag{14}$$

Once we have expressions for electric and magnetic energy within a sphere, it is possible to determine the expression for the total time-averaged EM energy inside the scatterer: $W(a) = W_E(a) + W_H(a)$. Explicitly,

$$\begin{aligned} W(a) = \frac{3}{4} W_0 \sum_{n=1}^{\infty} |\psi_n(y)|^{-2} \left[n\beta_n \mathcal{I}_{n+1}(y) \right. \\ \left. + (n+1)\beta_n \mathcal{I}_{n-1}(y) + (2n+1)\alpha_n \mathcal{I}_n(y) \right] , \end{aligned} \tag{15}$$

where

$$\alpha_n = |\psi_n(y)|^2 \left[\mathrm{Re}(m\widetilde{m}) |c_n|^2 + \mathrm{Re}\left(m\widetilde{m}^* \right) |d_n|^2 \right] , \tag{16}$$

$$\beta_n = |\psi_n(y)|^2 \left[\mathrm{Re}(m\widetilde{m}) |d_n|^2 + \mathrm{Re}\left(m\widetilde{m}^* \right) |c_n|^2 \right] . \tag{17}$$

Also, to obtain analogous expressions to the ones presented in [17], Eq. (15) can be rewritten as

$$\begin{aligned} W(a) \ = \ \frac{3}{4} W_0 \sum_{n=1}^{\infty} \frac{2n+1}{y^2 - y^{*2}} \left\{ \alpha_n \left[\frac{A_n(y^*)}{y} - \frac{A_n(y)}{y^*} \right] \right. \\ \left. + \beta_n \left[\frac{A_n(y^*)}{y^*} - \frac{A_n(y)}{y} \right] \right\} , \end{aligned} \tag{18}$$

with $y = mka$ and $A_n(y) = d_y \ln \psi(y)$.

C. Dielectric sphere

A particular situation to be considered here refers to a dielectric sphere, which has studied by Bott and Zdunkowski [17]. With this aim, consider the non-magnetic case, i.e., $\mu = \mu_1$. Thereby, it results that $m = \widetilde{m} = (\epsilon_1/\epsilon)^{1/2}$. With this assumption, note that $\mathrm{Re}\left(m^2\right) = (m^2 + m^{*2})/2$ and $\mathrm{Re}(mm^*) = |m|^2$. Substituting these into Eqs. (16) and (17), the expression for the internal energy obtained by Bott and Zdunkowski is recovered. Once again, we emphasize that our notation is not the same that is employed in [17]. Indeed, we can recover the same results by means of the following substitutions: $\xi_n(x) \dashrightarrow \zeta_n(x)$, $c_n \dashrightarrow md_n$, $d_n \dashrightarrow mc_n$, and assuming $m = \widetilde{m}$. Here, $\zeta_n(x) = x[j_n(x) - \imath y_n(x)]$ is the Ricatti-Hankel function of second kind, which is related to the choice of the time-harmonic dependence for the EM fields, like it is mentioned in the beginning of this description [6–10].

Employing the recurrence relations involving Bessel spherical functions [20, 21], we obtain the derivative of first order $A_n'(y) = -1 - A_n^2(y) + n(n+1)/y^2$. Therefore, using the L'Hospital rule, the limiting case of a perfect dielectric sphere, which takes place when $m_i \to 0$, provides $4y^2 \lim_{m_i \to 0} W(a) = 3W_0 \sum_{n=1}^{\infty} \gamma_n (2n+1)[1 + A_n^2(y) - n(n+1)/y^2]$, where $\gamma_n = m^2 |\psi_n(y)|^2 (|c_n|^2 + |d_n|^2)$. Unless some commented notation differences, this result is the same obtained in [17].

D. Absorption cross section

The classical Mie theory provides a set of useful expressions to calculate the scattering, total and absorption cross-sections in the scattering process. Explicitly, call σ_{sca} the scattering cross section and σ_{tot} the extinction (or total) cross section. Using the same framework of [6], one can write

$$\sigma_{\mathrm{sca}} \ = \ \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1) \left(|a_n|^2 + |b_n|^2 \right) \ , \tag{19}$$

$$\sigma_{\mathrm{tot}} \ = \ \frac{2\pi}{k^2} \sum_{n=1}^{\infty} (2n+1) \mathrm{Re}\left\{ a_n + b_n \right\} \ . \tag{20}$$

Consequently, the absorption cross section σ_{abs} associated with the scatterer is defined in terms of σ_{sca} and σ_{tot} by the relation: $\sigma_{\mathrm{abs}} = \sigma_{\mathrm{tot}} - \sigma_{sca}$. In other words, the absorption cross section in the Mie single scattering is determined by quantities and coefficients related only to the scattered EM fields [6, 7, 9]. Although it is suitable and even natural to express the absorption cross section in terms of the internal coefficients c_n and d_n, notice that we do not do any reference to the internal EM fields.

From the boundary conditions in the sphere problem [6], the Mie coefficients are linked by the equations below:

$$h_n^{(1)}(x) b_n \ = \ j_n(x) - j_n(mx) c_n \ , \tag{21}$$

$$h_n^{(1)}(x) a_n \ = \ j_n(x) - \widetilde{m} j_n(mx) d_n \ . \tag{22}$$

Thus, substituting the coefficients a_n and b_n into $\sigma_{\rm abs} = \sigma_{\rm tot} - \sigma_{sca}$ and manipulating that, we obtain the following expression respective to absorption cross section:

$$\sigma_{\rm abs} = \frac{2\pi}{k^2}\sum_{n=1}^{\infty}(2n+1)\Bigg\{\mathrm{Re}\left[\frac{\psi_n(mx)}{m\xi_n^*(x)}(c_n+\widetilde{m}d_n)\right] - \frac{|\psi_n(mx)|^2}{|m\xi_n(x)|^2}\left(|c_n|^2+|\widetilde{m}d_n|^2\right)\Bigg\}. \quad (23)$$

Finally, using the definition of c_n and d_n given by Eqs. (4) and (5) and the fact that $\mathrm{Re}\left[-i\xi_n^*(x)\xi_n'(x)\right] = \chi_n(x)\psi_n'(x) - \psi_n(x)\chi_n'(x) = 1$, where $\chi_n(x) = -xy_n(x)$ is the Ricatti-Neumann function, we obtain

$$\sigma_{\rm abs} = \frac{2\pi}{k^2}\sum_{n=1}^{\infty}(2n+1)\left(|c_n|^2+|d_n|^2\right) \times \mathrm{Im}\left[\frac{\widetilde{m}}{|m|^2}\psi_n(y)\psi_n'(y^*)\right]. \quad (24)$$

The exact expression (24) for the magnetic absorption cross section in terms of the internal Mie coefficients is not found in the classical books of the scattering theory [6, 7, 9] and it had not been determined so far.

IV. NUMERICAL RESULTS

In this section we present some numerical analysis from the exact equations determined in the sections above. Our aim is not to restrict our studies in some particular case of magnetic scattering, but to introduce a general formulation of the internal energy which can be used whether in magnetic case ($\mu \neq \mu_1$) or in nonmagnetic one ($\mu = \mu_1$). Here, all numerical results are obtained by means of a program created by us using the free software for scientific computation *Scilab* 5.1.1. As usual in the numerical Mie scattering, rather than to do infinite sums in $\sum_{n=1}^{\infty}$ in the calculation of the scattering quantities, which is impossible, we assume an approximation: finite sums with upper limit $n_{\max} = x + 4x^{1/3} + 2$, where x is the size parameter [8].

A. Normalized quantities

For numerical studies, it is suitable to define dimensionless quantities related to the internal energy and the Mie coefficients:

$$W_E^{\rm norm}(m,\widetilde{m},ka) = \frac{W_E(m,\widetilde{m},ka;\epsilon,a)}{W_0(\epsilon,a)}, \quad (25)$$

$$W_H^{\rm norm}(m,\widetilde{m},ka) = \frac{W_H(m,\widetilde{m},ka;\epsilon,a)}{W_0(\epsilon,a)}, \quad (26)$$

where W_E, W_H and W_0 are expressed by Eqs. (11), (14) and (13), respectively. The dependence of W_E and W_H on the quantity m^* is omitted.

Therefore, one can define the normalization of the total time-averaged internal energy by the relation $W_{\rm tot}^{\rm norm} = W_E^{\rm norm} + W_H^{\rm norm}$ or directly from Eq. (18): $W_{\rm tot}^{\rm norm}(m,\widetilde{m},ka) = W(a)/W_0$.

Also, from [21], we can obtain the recurrence relation $(2n+1)^2|j_n(\rho_1)|^2 = |\rho_1|^2\{|j_{n-1}(\rho_1)|^2 + |j_{n+1}(\rho_1)|^2 + 2\mathrm{Re}[j_{n-1}(\rho_1)j_{n+1}(\rho_1^*)]\}$; thus, Eq. (7) can be rewritten as

$$\frac{W_{rE}(a)}{W_0} = \frac{3}{4}\mathrm{Re}(m\widetilde{m})\sum_{n=1}^{\infty}\frac{n(n+1)}{2n+1}|d_n|^2 \times \Bigg\{\mathcal{I}_{n-1}(y) + \mathcal{I}_{n+1}(y) + \frac{2}{a^3}\int_0^a \mathrm{d}r r^2\,\mathrm{Re}\left[j_{n-1}(\rho_1)j_{n+1}(\rho_1^*)\right]\Bigg\}. \quad (27)$$

Note that the integral that appears in the sum above is quite similar to that one expressed in Eq. (A1). Although this one cannot be simplified by means of Eq. (A1), it is possible to show numerically that the result of this integral is proportional to a^3. It means that we can study the radial contribution to the internal energy normalized by W_0 using only the dimensionless parameters m, $\widetilde{m}$ and ka. The same argument can be applied to both $[W_{\theta E} + W_{\phi E}](a)/W_0$ and the analogous expressions respective to the internal magnetic field, given by Eqs. (8) and (10).

In the situations considered here, the values of $\mathrm{Re}(m\widetilde{m})$ and $\mathrm{Re}(m\widetilde{m}^*)$ are very close in such a way that $W_E(a) \approx W_H(a)$. Thus, we only consider the total time-averaged internal energy in our analysis. Further, we remark that although in these studies the relative magnetic permeability is assumed to be real, there is no such restriction in the calculated expressions. For soft microwave ferrites, a more realistic study should consider the magnetic loss.

Figs. 1 and 2 illustrate a succession of narrower picks of the values of the normalized internal energy in single magnetic Mie scattering as a function of the size parameter and relative magnetic permeability. Here, we are not concerned about to study in details the resonances picks and ripple structure due to the internal coefficients c_n and d_n [17, 22, 23]. The internal energy of a magnetic sphere presents resonances peaks even in the limit of small geometric size (compared to the wavelength). These resonances are due to the increase of the total cross section due to magnetism. This increase leads to a decrease of the photon mean free path in multiple scattering regime in a disordered system. The smaller mean free path favors the localization phenomenon as pointed in [3–5]. For dielectric spheres, these narrower resonance picks are well known and they are referred to as morphology dependent-resonances (MDRs) [24]. In the Mie theory, for large size parameters, these MDRs are commonly observed at the scattered and internal intensities and at the total cross section.

In our system, observe that when one increases the contribution of the magnetism in the scatterer, the values of the internal energy $W(a)$ become much larger than W_0, and the narrower picks appear even for small size

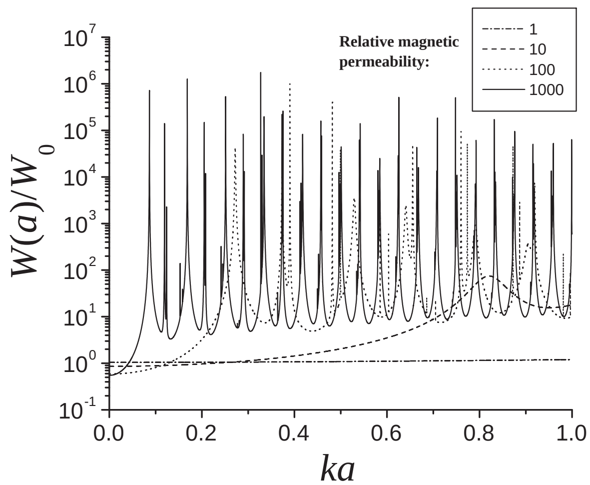

Figure 1: Comparison among the distributions of $W(a)/W_0$ in function of the size parameter ka. The values of the relative permittivity and permeability related to a non-absorptive sphere are $\epsilon_1/\epsilon = 1.4161$ and $\mu_1/\mu = 1, 10, 100, 1000$, respectively. The internal energy $W(a)/W_0$ is calculated in the interval $10^{-6} \leq ka \leq 1$, $\delta(ka) = 10^{-4}$.

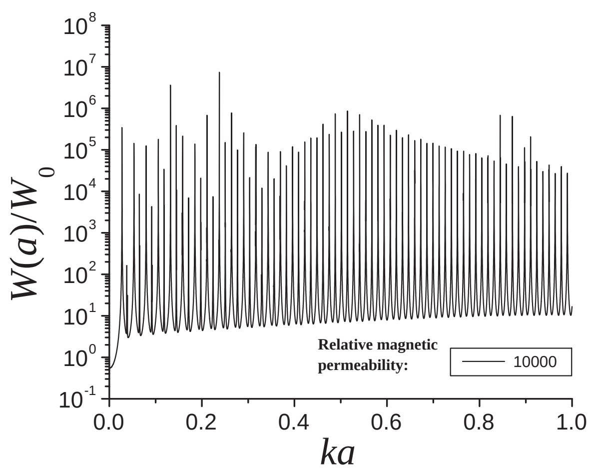

Figure 2: The normalized time-averaged internal energy $W(a)/W_0$ within a non-absorptive magnetic sphere plotted as a function of the size parameter ka. The values of the relative permittivity and permeability are $\epsilon_1/\epsilon = 1.4161$ and $\mu_1/\mu = 10^4$, respectively. The internal energy $W(a)/W_0$ is calculated in the interval $10^{-6} \leq ka \leq 1$, $\delta(ka) = 10^{-4}$.

parameters. In the nonmagnetic case reported in [17], there is an oppositive tendency, that is, both the internal energy and the absorption efficiency enhance with the size parameter. These difference between a nonmagnetic case and a magnetic one is illustrated in Fig. 3. This is due to the increase of the total cross section even though geometrically the scatterer is much smaller compared to the wavelength. In other words, the incident EM wave interacts strongly with the optical cross section instead of the geometrical one [2–5].

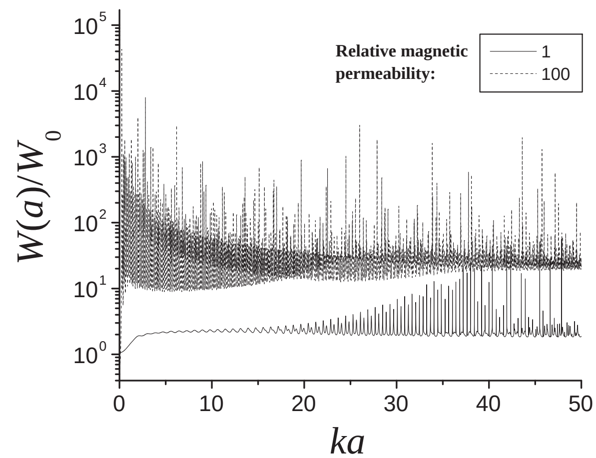

Figure 3: The normalized time-averaged internal energy $W(a)/W_0$ plotted as a function of the size parameter ka. The values of the relative permeability are $\mu_1/\mu = 1$ (nonmagnetic sphere) and $\mu_1/\mu = 100$ (magnetic sphere). The relative refraction index is $m = 1.334{+}1.5{\times}10^{-9}\imath$, which have been used in [17] in the nonmagnetic scattering approach. The quantities are calculated in the interval $1 \leq ka \leq 50$, $\delta(ka) = 0.01$.

It must be mentioned that this assumption of a non-absorptive magnetic diffusor is conditioned to the frequency range of the incident beam. Usually, it is controlled with an external static magnetic field [11–13, 15]. Indeed, there is a wide variety of soft ferrites which exhibits very large values of relative magnetic permeability at applied frequencies typically below 100 MHz with low magnetic loss [11]. For sake of simplicity and generality, the situations considered here do not take into account the scatterer in the magnetized state, and a scalar value for the μ_1/μ is adopted. The dependence on the angular frequency ω of the incident EM wave remains implicit on the value of the size parameter ka. Given a value of ω for the incident EM wave, a surrounding medium (ϵ, μ) and a scatter (ϵ_1, μ_1), one readily obtains $k = \omega(\mu\epsilon)^{1/2}$ and $k_1 = mk$ (see Fig. 4).

B. Weak absorption regime

In the weak absorption regime (wa), it is quite evident the relation of the time-averaged internal energy and the absorption efficiency when we compare them, as it is shown in the Fig. 5. This correlation between these quantities in nonmagnetic scattering have been studied in [17].

Analytically, for $m_i \ll m_r$ and $\widetilde{m}_i \ll \widetilde{m}_r$, we can write

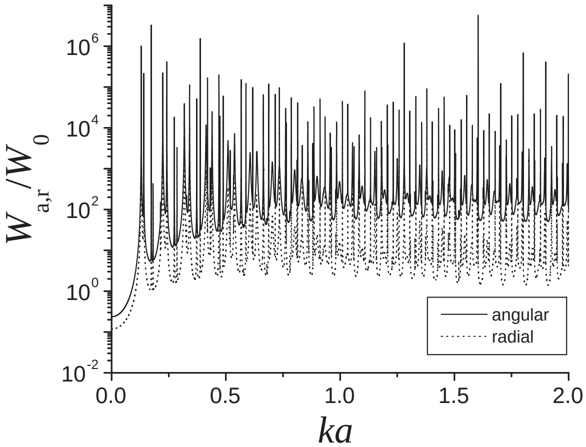

Figure 4: The separation of the total time-averaged internal energy in radial and angular contributions respective to both electric and magnetic fields. The values of the relative electric permittivity and magnetic permeability are $\epsilon_1/\epsilon = 10$ and $\mu_1/\mu = 100$, respectively. The component contributions $W_r(a)/W_0$ and $W_{\theta,\phi}(a)/W_0$ are calculated in the interval $10^{-6} \leq ka \leq 2$, $\delta(ka) = 10^{-4}$.

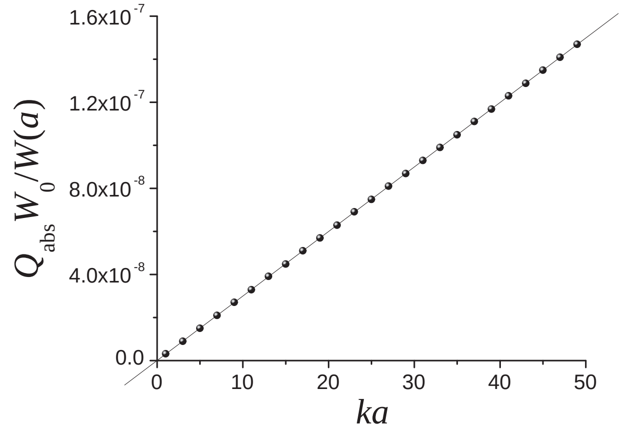

Figure 5: Ratio between the absorption efficiency $Q_{\rm abs}$ and the normalized time-averaged internal energy $W(a)/W_0$ plotted as a function of the size parameter ka. The values of the relative permeability and refraction index are $\mu_1/\mu = 1$ (nonmagnetic sphere) and $m = 1.334 + 1.5 \times 10^{-9}\imath$, respectively. The quantities are calculated in the interval $1 \leq ka \leq 49$, $\delta(ka) = 2$. The angular coefficient of linear regression is approximately 2.997×10^{-9}, which is in agreement with Eq. (29): $8m_i/(3m_r) \approx 2.998 \times 10^{-9}$.

$y^2 - y^{*2} \approx 4\imath x^2 m_r m_i$, $\mathrm{Re}(m\widetilde{m}) \approx m_r\widetilde{m}_r$ and $\mathrm{Re}\,(m\widetilde{m}^*) \approx m_r\widetilde{m}_r$. Using these approximations in Eq. (18), it follows that

$$W_{\rm wa}(a) \approx \frac{3}{8}W_0\frac{m_r}{m_i}\frac{2\widetilde{m}_r}{x^3 m_r^2}\sum_{n=1}^{\infty}(2n+1) \times \left(|c_n|^2 + |d_n|^2\right)\mathrm{Im}\left[\psi_n(y)\psi_n'(y^*)\right] \quad . (28)$$

Once the absorption efficiency in the Mie single scattering is defined by $Q_{\rm abs} = \sigma_{\rm abs}/\sigma_{\rm g}$, where $\sigma_{\rm g} = \pi a^2$ is the geometric cross section and $\sigma_{\rm abs}$ is expressed in Eq. (24), we can write that

$$W_{\rm wa}(a) \approx \frac{3}{8}W_0\frac{m_r}{m_i x}Q_{\rm abs} \ , \qquad (29)$$

which is the same relation obtained in [17] in the nonmagnetic case. Indeed, this approximation is valid wherever $m_i \ll m_r$ and $\widetilde{m}_i \ll \widetilde{m}_r$; it is not affected by the value of μ_1/μ.

In addition, for the case in which $m_r \approx 1$, one obtains $W_{\rm wa}(a) \approx W_0$. Therefore, in this particular situation, one can write $Q_{\rm abs} \approx 8x/(3m_i)$, which is a well-known expression [7, 17].

V. CONCLUSION

In this paper we generalize the exact expression of the time-averaged EM internal energy, obtained firstly in [17], to the case of magnetic spherical scatterers. Using the same framework of [6] and assuming the magnetic scattering approach [1], we determine analytical expressions for the contributions to the EM internal energy related to the fields components separately. The expressions for the EM internal energy within a dielectric sphere and the relation derived in [17] in the weak absorption regime between the internal energy and the absorption efficiency are recovered. In special, we find that the magnetism of the particle does not break the linear relation between the absorption efficiency and size parameter. To do so, we analytically calculate an expression for the absorption efficiency, which depends only on the internal magnetic Mie coefficients. In addition, we calculate the limiting cases of the magnetic Mie coefficients and present some important properties of the radial functions which are used to simply the obtained expressions. Finally, the main result of this work is that, even for small scatterers compared to the wavelength, the value of the EM internal energy within a magnetic sphere is much larger than that one associated with a sphere with the same properties of the surrounding medium. Physically, we ascribe this fact to the enhancement of the total cross section due to the magnetism in the scatterer.

Acknowledgements

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Appendix A: Special functions

1. Radial functions

For the situation in which $m_i \neq 0$ is verified, that is, the imaginary part of relative refraction index (absorptive component) is not null, we can write

$$\int_0^a r^2 \left| j_n(\rho_1) \right|^2 \mathrm{d}r = \frac{a^3 \left[y^* j_n(y) j_n'(y^*) - y j_n(y^*) j_n'(y) \right]}{y^2 - y^{*2}} = 2a^3 \left| j_n(y) \right|^2 \mathrm{Re} \left[\frac{\varphi_n(y^*)}{y^2 - y^{*2}} \right] , \quad \text{(A1)}$$

where $\rho_1 = mkr$, $y = mka$, $\varphi_n(y) = y \mathrm{d}_y [\ln \psi_n(y)]$. Eq. (A1) is provided, in terms of Bessel cylindrical functions, by Watson [20]. In the present context, to treat only with spherical Bessel functions, we have used the relation $(2\rho_1/\pi)^{1/2} j_n(\rho_1) = J_{n+1/2}(\rho_1)$ [21].

Also, if the relative refraction index m is real, accordingly to [20], the integral in Eq. (A1) can be simply rewritten as

$$\int_0^a r^2 j_n^2(\rho_1) \mathrm{d}r = \frac{a^3}{2} \left[j_n^2(y) - j_{n-1}(y) j_{n+1}(y) \right] , \quad \text{(A2)}$$

which is obtained by taking the limiting $m_i \to 0$ in Eq. (A1), and using L'Hospital rule and recurrence relations.

2. Angular functions

In the expansion of EM fields, it becomes natural to define the angular functions $\pi_n(\cos\theta) = P_n^1(\cos\theta)/\sin\theta$ and $\tau_n(\cos\theta) = \mathrm{d}P_n^1(\cos\theta)/\mathrm{d}\theta$, where θ is the scattering angle and P_n^1 is an associated Legendre function of first order. These angular functions are quite convenient in the calculation of fields intensities.

Due to properties involving the associated Legendre functions, π_n and τ_n satisfy the following expressions, $\forall n, n' \in \mathbb{N}$:

$$\int_{-1}^1 \mathrm{d}(\cos\theta) \left(\pi_n \pi_{n'} + \tau_n \tau_{n'} \right) = \frac{2n^2(n+1)^2}{2n+1} \delta_{n,n'} , \quad \text{(A3)}$$

$$\int_{-1}^1 \mathrm{d}(\cos\theta) \left(\pi_n \tau_{n'} + \tau_n \pi_{n'} \right) = 0 , \quad \text{(A4)}$$

$$\int_{-1}^1 \mathrm{d}(\cos\theta) \pi_n \pi_{n'} \sin^2\theta = \frac{2n(n+1)}{2n+1} \delta_{n,n'} . \quad \text{(A5)}$$

These expressions facilitate the determination of quantities involving fields intensities. We emphasize that, in the classical books of scattering theory, Eq. (A5) is not found in this explicit form [6, 7, 9].

Appendix B: Limiting cases

In these particular cases, we remark that $n = 1$ is sufficient to study the nonmagnetic scattering theory. Here, we imperatively have to consider $n = 1$ and $n = 2$ to keep consistent orders in the Mie coefficients.

1. Small particle limit

For the small argument limit into the Mie scattering coefficients, we obtain

$$a_1 \approx \frac{\imath x^3}{3} \frac{\varphi_1(mx) - 2m\widetilde{m}}{\varphi_1(mx) + m\widetilde{m}} - \frac{\imath x^5}{5} \frac{\left[\varphi_1(mx) - m\widetilde{m} \right]^2 - m\widetilde{m}\varphi_1(mx)}{\left[\varphi_1(mx) + m\widetilde{m} \right]^2} + \frac{x^6}{9} \left[\frac{\varphi_1(mx) - 2m\widetilde{m}}{\varphi_1(mx) + m\widetilde{m}} \right]^2 + \mathcal{O}(x^7) , \quad \text{(B1)}$$

$$b_1 \approx \frac{\imath x^3}{3} \frac{\varphi_1(mx) - 2m/\widetilde{m}}{\varphi_1(mx) + m/\widetilde{m}} - \frac{\imath x^5}{5} \frac{\left[\varphi_1(mx) - m/\widetilde{m} \right]^2 - (m/\widetilde{m})\varphi_1(mx)}{\left[\varphi_1(mx) + m/\widetilde{m} \right]^2} + \frac{x^6}{9} \left[\frac{\varphi_1(mx) - 2m/\widetilde{m}}{\varphi_1(mx) + m/\widetilde{m}} \right]^2 + \mathcal{O}(x^7) , \quad \text{(B2)}$$

$$a_2 \approx \frac{\imath x^5}{45} \frac{\varphi_2(mx) - 3m\widetilde{m}}{\varphi_2(mx) + 2m\widetilde{m}} + \mathcal{O}(x^7) , \quad \text{(B3)}$$

$$b_2 \approx \frac{\imath x^5}{45} \frac{\varphi_2(mx) - 3m/\widetilde{m}}{\varphi_2(mx) + 2m/\widetilde{m}} + \mathcal{O}(x^7) . \quad \text{(B4)}$$

For the Mie internal coefficients, the approximations assume the form below:

$$c_1 \approx \frac{mx^2}{\psi_1(mx)} \frac{m/\widetilde{m}}{\left[\varphi_1(mx) + m/\widetilde{m} \right]} \quad \text{(B5)}$$
$$- \frac{mx^4}{\psi_1(mx)} \frac{(m/\widetilde{m}) \left[\varphi_1(mx) - m/\widetilde{m} \right]}{2 \left[\varphi_1(mx) + m/\widetilde{m} \right]^2} + \mathcal{O}(x^5) , \quad \text{(B6)}$$

$$d_1 \approx \frac{(m/\widetilde{m})x^2}{\psi_1(mx)} \frac{m\widetilde{m}}{\left[\varphi_1(mx) + m\widetilde{m} \right]} \quad \text{(B7)}$$
$$- \frac{(m/\widetilde{m})x^4}{\psi_1(mx)} \frac{m\widetilde{m} \left[\varphi_1(mx) - m\widetilde{m} \right]}{2 \left[\varphi_1(mx) + m\widetilde{m} \right]^2} + \mathcal{O}(x^5) , \quad \text{(B8)}$$

$$c_2 \approx \frac{m}{3\psi_2(mx)} \frac{(m/\widetilde{m})x^3}{\left[\varphi_2(mx) + 2m/\widetilde{m} \right]} + \mathcal{O}(x^5) , \quad \text{(B9)}$$

$$d_2 \approx \frac{(m/\widetilde{m})}{3\psi_2(mx)} \frac{m\widetilde{m}x^3}{\left[\varphi_2(mx) + 2m\widetilde{m} \right]} + \mathcal{O}(x^5) . \quad \text{(B10)}$$

Note that, for these approximations, the scattering coefficients a_1, a_2, b_1 and b_2 have order $\mathcal{O}\left[(ka)^7 \right]$, whereas the internal coefficients c_1, c_2, d_1 and d_2 are $\mathcal{O}\left[(ka)^5 \right]$. Terms for $n > 2$ are ignored here.

2. Rayleigh approximation

In this approximation, in which $|m|x \ll 1$, the Mie scattering coefficients can be write as

$$\begin{aligned} a_1 &\approx \frac{-2\imath x^3}{3}\frac{m\widetilde{m}-1}{m\widetilde{m}+2} \\ &-\frac{\imath x^5}{5}\frac{m^3\widetilde{m}-6m\widetilde{m}+(m\widetilde{m})^2+4}{(m\widetilde{m}+2)^2} \\ &+\frac{4x^6}{9}\left(\frac{m\widetilde{m}-1}{m\widetilde{m}+2}\right)^2+\mathcal{O}(x^7)\ , \end{aligned} \quad \text{(B11)}$$

$$\begin{aligned} b_1 &\approx \frac{-2\imath x^3}{3}\frac{m/\widetilde{m}-1}{m/\widetilde{m}+2} \\ &-\frac{\imath x^5}{5}\frac{m^3\widetilde{m}-6m/\widetilde{m}+(m/\widetilde{m})^2+4}{(m/\widetilde{m}+2)^2} \\ &+\frac{4x^6}{9}\left(\frac{m/\widetilde{m}-1}{m/\widetilde{m}+2}\right)^2+\mathcal{O}(x^7)\ , \end{aligned} \quad \text{(B12)}$$

$$a_2 \approx \frac{-\imath x^5}{15}\frac{m\widetilde{m}-1}{2m\widetilde{m}+3}+\mathcal{O}(x^7)\ , \quad \text{(B13)}$$

$$b_2 \approx \frac{-\imath x^5}{15}\frac{m/\widetilde{m}-1}{2m/\widetilde{m}+3}+\mathcal{O}(x^7)\ , \quad \text{(B14)}$$

and the Mie internal coefficients assume the form

$$\begin{aligned} c_1 &\approx \frac{3}{2\widetilde{m}+m}\left[1+\frac{(mx)^2}{10}\right] \\ &-\frac{3x^2}{2}\left[1+\frac{(mx)^2}{10}\right]\frac{(2\widetilde{m}-m)}{(2\widetilde{m}+m)^2}+\mathcal{O}(x^5)\ , \end{aligned} \quad \text{(B15)}$$

$$\begin{aligned} d_1 &\approx \frac{3}{2+m\widetilde{m}}\left[1+\frac{(mx)^2}{10}\right] \\ &-\frac{3x^2}{2}\left[1+\frac{(mx)^2}{10}\right]\frac{(2-m\widetilde{m})}{(2+m\widetilde{m})^2}+\mathcal{O}(x^5)\ , \end{aligned} \quad \text{(B16)}$$

$$c_2 \approx \frac{5}{m\widetilde{m}(3+2m/\widetilde{m})}+\mathcal{O}(x^5)\ , \quad \text{(B17)}$$

$$d_2 \approx \frac{5}{m(3+2m\widetilde{m})}+\mathcal{O}(x^5)\ . \quad \text{(B18)}$$

Taking the particular case $m = \widetilde{m}$, Mie coefficients for nonmagnetic scattering are recovered [6].

3. Ferromagnetic limit for $x \ll 1$

This approximation, similar to Rayleigh limit, is derived directly from the approximation of small spheres compared to wavelength. Using the expressions for large argument limit present in [25], one can obtain

$$\begin{aligned} a_1 &\approx \frac{\imath x^3}{3}\frac{x\tan(mx)+2\widetilde{m}}{x\tan(mx)-\widetilde{m}} \\ &-\frac{\imath x^5}{5}\frac{[x\tan(mx)+\widetilde{m}]^2+\widetilde{m}x\tan(mx)}{[x\tan(mx)-\widetilde{m}]^2} \\ &+\frac{x^6}{9}\left[\frac{x\tan(mx)+2\widetilde{m}}{x\tan(mx)-\widetilde{m}}\right]^2+\mathcal{O}(x^7)\ , \end{aligned} \quad \text{(B19)}$$

$$\begin{aligned} b_1 &\approx \frac{\imath x^3}{3}\frac{\widetilde{m}x\tan(mx)+2}{\widetilde{m}x\tan(mx)-1} \\ &-\frac{\imath x^5}{5}\frac{[\widetilde{m}x\tan(mx)+1]^2+\widetilde{m}x\tan(mx)}{[\widetilde{m}x\tan(mx)-1]^2} \\ &+\frac{x^6}{9}\left[\frac{\widetilde{m}x\tan(mx)+2}{\widetilde{m}x\tan(mx)-1}\right]^2+\mathcal{O}(x^7)\ , \end{aligned} \quad \text{(B20)}$$

$$a_2 \approx \frac{\imath x^5}{45}\frac{x-3\widetilde{m}\tan(mx)}{x+2\widetilde{m}\tan(mx)}+\mathcal{O}(x^7)\ , \quad \text{(B21)}$$

$$b_2 \approx \frac{\imath x^5}{45}\frac{\widetilde{m}x-3\tan(mx)}{\widetilde{m}x+2\tan(mx)}+\mathcal{O}(x^7)\ ; \quad \text{(B22)}$$

the internal coefficients are

$$\begin{aligned} c_1 &\approx \frac{m}{\cos(mx)}\frac{x^2}{[\widetilde{m}x\tan(mx)-1]} \\ &-\frac{mx^4}{2\cos(mx)}\frac{[\widetilde{m}x\tan(mx)+1]}{[\widetilde{m}x\tan(mx)-1]^2}+\mathcal{O}(x^5)\ , \end{aligned} \quad \text{(B23)}$$

$$\begin{aligned} d_1 &\approx \frac{m}{\cos(mx)}\frac{x^2}{[x\tan(mx)-\widetilde{m}]} \\ &-\frac{mx^4}{2\cos(mx)}\frac{[x\tan(mx)+\widetilde{m}]}{[x\tan(mx)-\widetilde{m}]^2}+\mathcal{O}(x^5)\ , \end{aligned} \quad \text{(B24)}$$

$$c_2 \approx \frac{-m}{3\cos(mx)}\frac{x^3}{[2\tan(mx)+\widetilde{m}x]}+\mathcal{O}(x^5)\ , \quad \text{(B25)}$$

$$d_2 \approx \frac{-m}{3\cos(mx)}\frac{x^3}{[2\widetilde{m}\tan(mx)+x]}+\mathcal{O}(x^5)\ . \quad \text{(B26)}$$

In this case, since low order in the size parameter is used, one can obtain an analytical expression for the physical quantities, such as cross sections, for instance.

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